A Nonlinear Response Amplitude Operator for Maritime Dynamical Systems, Offshore Applications, and Wave Energy Converters

Georgios D. Gkikas
Maritime Research Institute Netherlands (MARIN)
Wageningen, The Netherlands

A nonlinear response amplitude operator (NRAO) is derived that is able to model a wide class of nonlinear systems and structures usually found in maritime and offshore applications. The corresponding functional scheme is of the Volterra series type, and its order is explicitly dictated by the order of nonlinearity of the actual dynamical system. It is outlined that the proposed operator is not bounded by any amplitude or frequency constraints and it can estimate the response of a nonlinear system even for multichromatic excitations that consist of modes with significantly different (or not) amplitudes and frequencies.

INTRODUCTION

According to standard maritime engineering practice, in order to interpret the dynamic response of a floating body or an offshore structure with respect to the surrounding wave excitations, a frequency transfer function, also termed "response amplitude operator" (RAO), is derived and used. The validity of this operator is based on the assumption of linearity between the excitation and the system's response. A major advantage that comes with the analysis of dynamical systems in a linear context is that we can employ simple harmonic excitations in order to obtain the required operator from a straightforward input/output relation and without any further computational effort.

However, in low- or high-dimensional nonlinear systems analysis, things can be far more complicated, and the approximation functionals or their equivalent systemic structures may not be able to successfully model the inherent nonlinearities of the actual system. In many cases the test input, used for the estimation of the nonlinear system's elements (i.e., integral kernels or static polynomials), needs to be bichromatic, multichromatic, or white Gaussian noise. Furthermore, the derivation of the resulting generalized transfer functions is based on tedious algorithms accompanied by expensive computational schemes that make continuous use of averaging techniques for any low- and higher-order auto and cross, input/output correlations. Methods that employ harmonic inputs are usually fit for narrow band applications, although they are also quite constrained with respect to the excitation's amplitude range. Furthermore, the target system needs to exhibit sufficiently smooth nonlinearities in the sense that the ratios between the harmonics' magnitudes should not change significantly as we vary the excitation's amplitude or frequency. Because the Volterra series is essentially the functional extension of the Taylor series, it should be noted that nonsmooth nonlinearities cannot be tackled unless the order of terms becomes too high. However, it could be said that Preisach-type operators could further be appended to the proposed scheme in order to cover to a certain extent such complex nonlinear behaviors, while discontinuous dynamical systems, such as the thermodynamic system presented in this work, can already be dealt with adequately. It is further noted that the structure of the proposed method, i.e., no amplitude or frequency range restrictions, could provide us with the ability to tackle problems with saturation dynamics as well.

The concept of a functional representation of a system of nonlinear differential equations through a Volterra series was created on the basis of a generalization of power series solutions by Volterra (1959) and initially applied to nonlinear systems by Wiener (1942). The Volterra series approach, being a power series generalization, is related to convergence issues, as shown by Palm and Poggio (1977). Consequently, Boyd et al. (1984) provided the conditions for the series to converge with the aid of various versions of the gain bound theorem, while Boyd and Chua (1985) derived, via the fading memory concept, suitable approximation theorems for a truncated Volterra series defined on noncompact subsets of the input space, i.e., defined on the infinite line. Some very useful theoretical results with respect to classes of nonlinear dynamical systems that admit Volterra series modeling and identification were among others derived by Palm and Poggio (1977), Schetzen (1980), Rugh (1981), and Palm (1978). In the latter work in particular, equivalency conditions between different types of nonlinear systemic approximation schemes were derived, and one of the main results of Palm (1978), also fully exploited in this work, is that all continuous-time dynamical systems can be approximated by the class of separable-kernel polynomial systems, a subclass of polynomial systems. It is noted that the term "separable kernel" corresponds to the ability to express an N-dimensional kernel as an N-product of one-dimensional (linear) kernels. However, even with such a sound theoretical background in the capabilities of nonlinear systems identification, the development of a widely applicable, generic approach seems to be lacking; we can only develop techniques that apply to specific characteristics of the nonlinear systems as shown by Barrett (1963), Boyd and Chua (1985), Storer (1991), Spanos and Donley (1991, 1992), Kareem et al. (1995), Donley and Spanos (1992), Van de Wouw et al. (2002), Chatterjee and Vyas (2003, 2004), Lang and Billings (1997), Doyle et al. (2002), Ogunfumi (2007), and Giri and Bai (2010).

In an effort to overcome all of the above issues, a Volterra series-based nonlinear response amplitude operator (NRAO) is proposed. The order of the functional scheme is directly dependent on the order of the nonlinearity of the actual system, while the Volterra kernels become implicitly dependent on the input since the oscillatory properties of the excitation are used as parameters for the determination of the final form of the nonlinear operator. The