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Wave-current Interaction in Shallow Flows

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ABSTRACT

In this paper, a time domain numerical method to investigate wave-current interaction in shallow flows is proposed. In the framework of Helmholtz decomposition, the viscous component is described by a depth-averaged model, and free-surface effects are considered by a fully 3-D potential linear model. The basic hypothesis of the proposed approach is that all the interaction effects can be described by means of a potential flow field. The depth-averaged, unsteady, viscous flow is obtained by using a suitable eddy viscosity model, within the RANS framework, and the numerical solution is achieved by a finite difference method. On the other hand, the potential, fully 3-D flow field is computed by means of a boundary integral formulation.

In several situations, free-surface flows are characterized by wave-current interaction phenomena. Jets and streams can be highly affected by waves, while wave patterns can be significantly modified by currents as well. In practical coastal engineering applications, characterized by large space and time scales, these very complicated features are often tackled by means of the classical radiation stresses approach, proposed by Longuet-Higgins and Stewart (1964) in their milestone paper; in this case, the steady streaming flow components of wave motion are taken into account. Mild-slope methods are also widely used to predict wave propagation in coastal regions (Berkhoff, 1972; Ito and Tanimoto, 1972; Gao and Radder, 1998). In a more recent paper (Teng et al., 2001) a fully 3-D potential flow model for the investigation of wave diffraction over a shoal- and wave-current interaction is proposed. This approach is strictly linear, since the Green function involved in the integral formulation fulfills linear free- surface boundary conditions; in addition, in this work wave-current interaction is restricted to the uniform stream case. In several situations, however, nonuniform viscous flow features are involved, as in the case of wave-jet interaction.

A fully nonlinear approach for 3-D, unsteady, potential free-surface flows was proposed by Lalli et al. (1996) and Lalli (1997); in these papers the velocity potential is split into incident and perturbation components (Landrini, 1994) and represented by means of a simple layer distribution over the boundaries of the fluid domain (Bassanini et al., 1994). Starting from these results, in the present work a numerical method for the investigation of the hydrodynamic problem of wave-current interaction in the time domain is proposed. The fluid density is assumed to be uniform. Let $\underline{u}(x, y, z, t)$ be an unsteady, rotational velocity field, which according to Helmholtz decomposition can be written as:

$$\underline{u}(x, y, z, t) = \underline{\bar{u}}(x, y, t) + \nabla\Phi(x, y, z, t) \tag{1}$$

$$\Gamma(x, y, t) = \bar{\eta}(x, y, t) + \eta(x, y, t) + \tilde{\eta}(x, y, t) \tag{2}$$

where Γ = free-surface elevation; $\underline{\bar{u}} = (\bar{u}, \bar{v})$ = depth-averaged rotational flow (velocity); $\bar{\eta}$ = depth-averaged rotational flow (free-surface elevation); $\nabla\Phi = \nabla\varphi + \nabla\tilde{\varphi}$ = fully 3-D potential flow; φ = incoming wave (velocity potential); η = incoming wave (free-surface elevation); $\tilde{\varphi}$ = wave-structure-current interaction (velocity potential); and $\tilde{\eta}$ = wave-structure-current interaction (free-surface elevation).

Thus, in the present approach, viscous effects are described by a depth-averaged model, while free-surface effects are considered by a fully 3-D potential model. Indeed, if the flow features (see e.g. Clercx et al., 2003) can allow a 2-D approach for the viscous component, wave propagation is typically 3-D. In particular, as is well known, pressure distribution along the vertical in the presence of a wave train cannot be assumed to be hydrostatic.

In the case of small amplitude, the incoming wave potential φ is given by the solution of the problem:

$$\frac{\partial\varphi}{\partial t} + g\eta = 0 \qquad \text{at } z = 0 \tag{3}$$

$$\frac{\partial\eta}{\partial t} = \frac{\partial\varphi}{\partial z} \qquad \text{at } z = 0 \tag{4}$$

$$\frac{\partial\varphi}{\partial z} = 0 \qquad \text{at } z = -h_0 \tag{5}$$

The incoming wave is then expressed as a combination of linear waves. Depth variations in the fluid domain and any kind of obstacles can be taken into account by means of the perturbation potential $\tilde{\varphi}$. Extension to the nonlinear formulation is straightforward (Lalli et al., 1996; Lalli, 1997), though not trivial from the computational point of view. In this case, the thorny problems are concerned with wave breaking. It is worth noting that the present approach does not allow, in principle, wave breaking simulation (see e.g. Guignard et al., 1999), since breaking gives rise to fully

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